INVESTIGATE RULES OF GAMES

Practice 2

Practice 2 – Suggested solution (Case 1)

Objective of the task:

To investigate and generate rules for the probabilities of all possible differences between the two numbers if you select two random cards (from 2 sets of 4 consecutive numbers).

Solution (Case 1: the 2 sets of cards are the same, we draw 1 card from each set, and we only want positive difference):

Level 1 -2:

I am going to use a table to find all possible outcomes when drawing two cards randomly (from 2 sets of 4 consecutive numbers, says: 1,2,3,4,1,2,3,4).

1st card / 2nd card	1	2	3	4
1	(1,1) 0	(1,2) 1	(1,3) 2	(1,4) 3
2	(2,1) 1	(2,2) 0	(2,3) 1	(2,4) 2
3	(3,1) 2	(3,2) 1	(3,3) 0	(3,4) 1
4	(4,1) 3	(4,2) 2	(4,3) 1	(4,4) 0
combination / difference				

I am going to do an experiment to find the experimental probabilities of each event.

Difference of the two cards drawn (from 2 set of 4 consecutive numbers)	**Tally**	**Frequency**	**Experimental Probability** $= \frac{\text{number of times the event occurred}}{\text{total number of trials}}$
0	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 |	51	$\frac{51}{200} = 0.255$
1	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 ||||	79	$\frac{79}{200} = 0.395$
2	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 ||	47	$\frac{47}{200} = 0.235$
3	𝍸 𝍸 𝍸 𝍸 |||	23	$\frac{23}{200} = 0.115$
	Total:	200	1

I have drawn 2 cards randomly and find their differences for 200 times and recorded the above results.

Prediction-

According to the experimental probabilities, I predict that if I have done 400 times instead of 200, I will get around 100 trials to be "0" as the two cards' difference.

Level 3-4:

Suggest how these patterns happen to other similar case-

If the numbers of the cards get bigger, says: 5,6,7,8,5,6,7,8

I predict that the probabilities of the corresponding events will not change, as I can see from the following table, the final difference is still 0, 1, 2, 3, the distribution of the probabilities are similar to case 1 shown above.

1st card / 2nd card	5	6	7	8
5	(5,5) 0	(5,6) 1	(5,7) 2	(5,8) 3
6	(6,5) 1	(6,6) 0	(6,7) 1	(6,8) 2
7	(7,5) 2	(7,6) 1	(7,7) 0	(7,8) 1
8	(8,5) 3	(8,6) 2	(8,7) 1	(8,8) 0
combination / difference				

Level 5-6:
I am going to find and explain the rules and patterns from the experimental results.

From the above experimental results and probabilities, I find out that:

(i) There are 16 possible combinations of getting 2 cards and only 4 different events for the differences of the number of the two cards, which is 0, 1, 2, 3, in my case.
(ii) There is the most chance to get a 1, P (two cards' difference is 1), which is 0.395 from my experiment.
(iii) There is the least chance to get a 3, P (two cards' difference is 3), which is only 0.115 from my experiment.
(iv) The chance of getting a 0 or 2 is very close to each other and I predict they should be the same if I did enough trials. The experimental probabilities are P (two cards' difference is 0), and P (two cards' difference is 2) is 0.255 and 0.235 respectively.
(v) Getting P (two cards' difference is 1) will be 3 times of getting P (two cards' difference is 1)
(vi) The total of the experimental probabilities of all the outcomes is 1.

Test some of the pattern-
I can test one of the above rule is correct by finding its theoretical probabilities.

Since there are 16 difference outcomes and only 2 of them is "3"
therefore the theoretical probability P (two cards' difference is 3) $=\frac{2}{16}=\frac{1}{8}=0.125$
which is quite close the experimental probabilities 0.115.

Level 7-8
Test/Justify my rules/patterns are correct by finding all of the theoretical probabilities (giving more examples)

Difference of the two cards drawn (from 2 set of 4 consecutive numbers)	**Theoretical Probability** $=\frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}}$	**Experimental Probability** $=\frac{\text{number of times the event occurred}}{\text{total number of trials}}$
0	$\frac{4}{16}=0.25$	$\frac{51}{200}=0.255$
1	$\frac{6}{16}=0.375$	$\frac{79}{200}=0.395$
2	$\frac{4}{16}=0.25$	$\frac{47}{200}=0.235$
3	$\frac{2}{16}=0.125$	$\frac{23}{200}=0.115$
Total	1	1

By comparing the experimental probabilities and the theoretical probabilities:

(i) There are 16 possible combinations of getting 2 cards and only 4 different events for the differences of the number of the two cards, which is 0, 1, 2, 3, in my case (see table under level 1-2.
(ii) There is the most chance to get a 1, P (two cards' difference is 1), which is 0.375 from the above table, which is very close to my experiment result, 0.395.
(iii) There is the least chance to get a 3, P (two cards' difference is 3), which is only 0.125 theoretically and is very close to my experimental result 0.115.
(iv) The chance of getting a 0 or 2 is the same, P (two cards' difference is 0) and P (two cards' difference is 2), theoretically is both 0.25, which prove my rules, patterns and predictions are correct since both experimental probability 0.255 and 0.235 are very close to 0.25.
(v) Getting P (two cards' difference is 1) =0.375 = 0.125×3 which is 3 times of getting P (two cards' difference is 1)=0.125
(vi) The total of the experimental probabilities of all the outcomes is 1 (0.25+0.25+0.125+0.375=1).

Conclusions:

After going through the whole process of investigation and justification by finding the experimental and theoretical probabilities, I conclude that my rules and patterns for the probabilities of the difference by drawing 2 cards randomly from 2 sets of 4 consecutive cards are correct.

(i) There are 16 possible combinations if the two sets of card are the same, and the four differences are 0, 1, 2, 3,

(ii) While the second one (difference of the two cards is 1) will have the greatest probability and,

(iii) The last one (difference of the two cards is 3) will have the smallest probability,

(iv) The first number and third number (0 and 2) have the same probability,

(v) The chance of getting the difference as "1" is 3 times as the chance of getting the "3",

(vi) Probability of getting all of the 4 events' added up to be 1.